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Polarization and spin effects in neutralino production and decay

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Abstract. We study the production of neutralinos $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_j$ with polarized beams and the subsequent decays $\tilde{\chi}^0_i \to \tilde{\chi}^0_k\ell^+\ell^-$ and $\tilde{\chi}^0_j \to \tilde{\chi}^0_l\ell^+\ell^-$, including the complete spin correlations between production and decay. We present analytical formulae for the differential cross section of the combined process of production and decay of neutralinos. We also allow for complex couplings. The spin correlations have a strong influence on the decay angular distributions and the corresponding forward–backward asymmetries. They are very sensitive to the SUSY parameters and depend strongly on the beam polarizations. We present numerical results for the cross section and the electron forward–backward asymmetry for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$, $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 e^+e^-$. We study the dependence on the parameter M' for various mass splittings between $\tilde{e}_L$ and $\tilde{e}_R$ and different beam polarizations.

1 Introduction

The search for supersymmetric (SUSY) particles is one of the main goals of present and future colliders. In particular, an e^+e^- linear collider will be an excellent discovery machine for SUSY particles [1]. Experiments at a linear collider will also allow us to determine precisely the parameters of the underlying SUSY model [2].

The neutralinos, the supersymmetric partners of the neutral gauge and Higgs bosons, are of particular interest as they are expected to be relatively light. Most studies of neutralino production $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_j$, i, j=1,…,4, and decays have been performed in the Minimal Supersymmetric Standard Model (MSSM) (see [3–5], and references therein). For a clear identification of neutralinos a precise analysis of their decay characteristics is indispensable. By measuring cross sections, branching ratios, various energy and angular distributions of the decay products of the neutralinos, one obtains valuable information about the parameters of the MSSM.

Since decay angular distributions depend on the polarization of the parent particles one has to take into account the spin correlations between production and decay of the neutralinos. In general, quantum mechanical interference effects between various polarization states of the decaying particles preclude simple factorization of the differential cross section into a production and a decay factor [6, 7], unless the production amplitude is dominated by a single spin component [8]. A variety of event generators for production and decay of SUSY particles, such as DFGT, SUSYGEN, GRACE and CompHEP [9], have been developed which include spin correlations between production and decay.

In a previous paper [10] the process $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_j$, i, j=1,…,4, with unpolarized beams and the subsequent leptonic decays $\tilde{\chi}^0_i \to \tilde{\chi}^0_k\ell^+\ell^-$, $\tilde{\chi}^0_j \to \tilde{\chi}^0_l\ell^+\ell^-$ have been studied with complete spin correlations. Some results for polarized beams have been presented in [11]. In the present paper we give the complete analytical formulae for polarized beams. We fully include the spin correlations between production and decay. The formulae are given in a transparent form in the laboratory system (which is identical to the overall CMS) in terms of the basic kinematic variables. Moreover, we include complex couplings allowing for studies of CP violating phenomena. The expression for the differential cross section is composed of the joint spin density matrix for the production of neutralinos and the decay matrices for their leptonic decays. Our formulae can easily be extended to hadronic decays.

The masses and couplings of the neutralinos depend on the MSSM parameter M', M, μ and $\tan\beta$. The parameters M, μ and $\tan\beta$ can in principle be determined by chargino production alone [12, 13]. The cross section for chargino production with polarized beams and the decay angular distributions also give information on the sneutrino mass $m_{\tilde{\nu}}$ [14]. However, a precise determination of the parameter M' is only possible in the neutralino sector. A study of neutralino production and decay also gives information about the masses of the left and right selectrons, $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$.

It is known that the forward–backward asymmetry of the production process $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_j$ vanishes due to the Majorana nature of the neutralinos [3,15,16]. However, taking into account neutralino decay, for instance $\tilde{\chi}^0_i \to \ell^+\ell^-\tilde{\chi}^0_k$, the forward–backward asymmetry A_{FB} of one of the decay leptons does not vanish [10,11]. This is a consequence of spin correlations between production and decay. As we shall show this A_{FB} depends very sensitively on the SUSY parameters. Furthermore, it depends very strongly on the beam polarization. The forward–backward asymmetry A_{FB} of the decay lepton is due to a complex interplay of the Z and $\tilde{\ell}_L$, $\tilde{\ell}_R$ exchange amplitudes in production and decay, where the polarization of $\tilde{\chi}^0_i$ plays a crucial rôle. The polarization vector of $\tilde{\chi}^0_i$ is determined by the characteristics of the production process and strongly influences the decay distribution.

The main purpose of our paper is the presentation of the formulae for the combined process $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_j$, $\tilde{\chi}^0_i \to \ell^+\ell^-\tilde{\chi}^0_k$ and $\tilde{\chi}^0_j \to \ell^+\ell^-\tilde{\chi}^0_l$, with both beams polarized. We also present numerical results for the cross section and the lepton forward–backward asymmetry A_{FB} of $e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_1$, $\tilde{\chi}^0_2 \to \ell^+\ell^-\tilde{\chi}^0_1$. In particular, we study their dependence on M', $m_{\tilde{\ell}_L}$ and $m_{\tilde{\ell}_R}$, and on the beam polarization.

In Sect. 2 we present the formalism used. In Sect. 3 we give the formulae for the spin density production matrix of the neutralinos in the laboratory system for polarized beams. In Sect. 4 we give the decay matrices for the leptonic decay of $\tilde{\chi}^0_i$ and $\tilde{\chi}^0_j$ in covariant form. In the Sect. 5 we present our numerical results for the cross section and the forward–backward asymmetry A_{FB} of the decay lepton as a function of the parameter M' for various slepton masses $m_{\tilde{\ell}_L}, m_{\tilde{\ell}_R}$ and for different beam polarizations.

2 Definitions and formalism

We give the analytical formulae for the differential cross section of neutralino production

$$e^-(p_1) + e^+(p_2) \to \tilde{\chi}^0_i(p_3) + \tilde{\chi}^0_j(p_4), \tag{1}$$

with polarized beams and the subsequent leptonic decays

$$\tilde{\chi}^0_i(p_3) \to \tilde{\chi}^0_k(p_5) + \ell^+(p_6) + \ell^-(p_7), \tag{2}$$
$$\tilde{\chi}^0_j(p_4) \to \tilde{\chi}^0_l(p_8) + \ell^+(p_9) + \ell^-(p_{10}), \tag{3}$$

with complete spin correlations between production and decay.

2.1 Lagrangian and couplings

The parts of the interaction Lagrangian of the MSSM relevant for our study are (in our notation and conventions we follow closely [17]):

$$\mathcal{L}_{Z^0\ell^+\ell^-} = -\frac{g}{\cos\theta_W} Z_\mu \bar{\ell}\gamma^\mu[L_\ell P_L + R_\ell P_R]\ell, \tag{4}$$

$$\mathcal{L}_{Z^0\tilde{\chi}^0_i\tilde{\chi}^0_j} = \frac{1}{2}\frac{g}{\cos\theta_W} Z_\mu \bar{\tilde{\chi}}^0_i\gamma^\mu[O''^L_{ij} P_L + O''^R_{ij} P_R]\tilde{\chi}^0_j, \tag{5}$$

$$\mathcal{L}_{\ell\tilde{\ell}\tilde{\chi}^0_i} = g f^L_{\ell i}\bar{\ell}P_R\tilde{\chi}^0_i\tilde{\ell}_L + g f^R_{\ell i}\bar{\ell}P_L\tilde{\chi}^0_i\tilde{\ell}_R + \text{h.c.}, \quad i,j = 1,\cdots,4. \tag{6}$$

The couplings are:

$$L_\ell = T_{3\ell} - e_\ell\sin^2\theta_W, \quad R_\ell = -e_\ell\sin^2\theta_W, \tag{7}$$

$$f^L_{\ell i} = -\sqrt{2}\left[\frac{1}{\cos\theta_W}(T_{3\ell} - e_\ell\sin^2\theta_W)N_{i2} + e_\ell\sin\theta_W N_{i1}\right],$$
$$f^R_{\ell i} = -\sqrt{2}e_\ell\sin\theta_W\left[\tan\theta_W N^*_{i2} - N^*_{i1}\right], \tag{8}$$

$$O''^L_{ij} = -\frac{1}{2}\Big(N_{i3}N^*_{j3} - N_{i4}N^*_{j4}\Big)\cos 2\beta - \frac{1}{2}\Big(N_{i3}N^*_{j4} + N_{i4}N^*_{j3}\Big)\sin 2\beta,$$
$$O''^R_{ij} = -O''^{L*}_{ij}, \text{ with } i,j = 1,\ldots,4. \tag{9}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, g is the weak coupling constant ($g = e/\sin\theta_W$, $e > 0$), and e_ℓ and $T_{3\ell}$ denote the charge and the third component of the weak isospin of the lepton ℓ, $\tan\beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two neutral Higgs fields. N_{ij} is the unitary 4×4 matrix which diagonalizes the neutral gaugino-higgsino mass matrix $Y_{\alpha\beta}$, $N_{i\alpha}Y_{\alpha\beta}N_{k\beta} = m_{\tilde{\chi}^0_i}\delta_{ik}$. We use the basis $\tilde{\gamma}, \tilde{Z}, \tilde{H}^0_a, \tilde{H}^0_b$ [3].

2.2 CP conserving and CP violating case

In the formulae for the cross section we shall present in the following, one has to distinguish between two cases, CP conservation and CP violation: If CP is conserved the neutralino mass matrix $Y_{\alpha\beta}$ is real and the matrix N_{ij} can be chosen real and orthogonal. Then all the couplings given in (8), (9) are real. In this case some of the mass eigenvalues may be negative. We therefore write the eigenvalues in the form $m_{\tilde{\chi}^0_i} = \eta_i m_i$, $i = 1,\ldots,4$, with $m_i \geq 0$ and $\eta_i = \pm1$ [3]. η_i is related to the CP eigenvalue of the neutralino $\tilde{\chi}^0_i$ [18].

If CP is violated the neutralino mass matrix is complex and the matrix N_{ij} is complex and unitary. In this case the diagonalization of the mass matrix is done with the singular value decomposition method, $N_{i\alpha}Y_{\alpha\beta}N_{k\beta} = m_i\delta_{ik}$, $m_i \geq 0$. In this method all masses m_i are chosen positive. The neutralino couplings, (8), (9), are complex.

The formulae given below are applicable to both cases. In the case of CP conservation the imaginary parts of all couplings are zero and the sign η_i of the mass eigenvalues, appearing explicitly in the formulae, has to be taken into account.

In the case of CP violation the imaginary parts of the couplings do not vanish. All factors η_i appearing in the formulae have to be set $\eta_i = +1$.

2.3 Helicity amplitudes and cross section

For the calculation of the amplitude of the combined processes of neutralino production and decays, (1)–(3), we

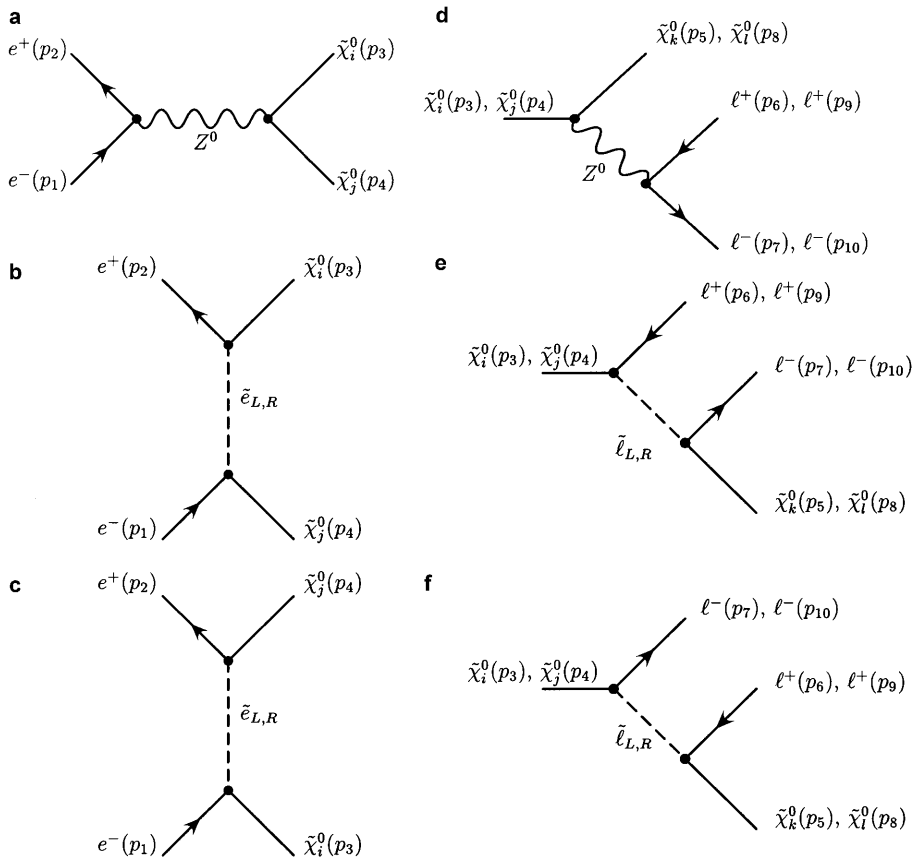

Fig. 1a–f. Feynman diagrams for the s,t,u channel of the production process, $e^+e^- \to \tilde\chi^0_i\tilde\chi^0_j$, and the s_i,t_i,u_i and s_j,t_j,u_j channels of the decay processes $\tilde\chi^0_i \to \tilde\chi^0_k\ell^+\ell^-$ and $\tilde\chi^0_j \to \tilde\chi^0_l\ell^+\ell^-$.

use the same formalism as for the chargino production and decays [13], following the method of [19]. The helicity amplitudes for the production (1) are

$$T_P^{\lambda_i\lambda_j} = T_P^{\lambda_i\lambda_j}(s,Z) + T_P^{\lambda_i\lambda_j}(t,\tilde e_L) + T_P^{\lambda_i\lambda_j}(t,\tilde e_R) + T_P^{\lambda_i\lambda_j}(u,\tilde e_L) + T_P^{\lambda_i\lambda_j}(u,\tilde e_R), \quad (10)$$

and those for the decays, (2) and (3) are

$$T_{D,\lambda_i} = T_{D,\lambda_i}(s_i,Z) + T_{D,\lambda_i}(t_i,\tilde\ell_L) + T_{D,\lambda_i}(t_i,\tilde\ell_R) + T_{D,\lambda_i}(u_i,\tilde\ell_L) + T_{D,\lambda_i}(u_i,\tilde\ell_R), \quad (11)$$

$$T_{D,\lambda_j} = T_{D,\lambda_j}(s_j,Z) + T_{D,\lambda_j}(t_j,\tilde\ell_L) + T_{D,\lambda_j}(t_j,\tilde\ell_R) + T_{D,\lambda_j}(u_j,\tilde\ell_L) + T_{D,\lambda_j}(u_j,\tilde\ell_R). \quad (12)$$

They correspond to the Feynman diagrams in Fig. 1, and are given in the Appendix A, (A.1)–(A.12).

We introduce the kinematic variables $s = (p_1+p_2)^2$, $t = (p_1-p_4)^2$, and $u = (p_1-p_3)^2$ for the production process, (1), and $s_i = (p_6+p_7)^2$, $t_i = (p_3-p_6)^2$, $u_i = (p_3-p_7)^2$ for the decay process of the neutralino $\tilde\chi^0_i$, (2), and $s_j = (p_9+p_{10})^2$, $t_j = (p_4-p_9)^2$ and $u_j = (p_4-p_{10})^2$ for the decay of the neutralino $\tilde\chi^0_j$, (3), with the particle momenta p_k as denoted in (1)–(3).

The amplitude for the whole process is

$$T = \Delta(\tilde\chi^0_i)\Delta(\tilde\chi^0_j)\sum_{\lambda_i,\lambda_j} T_P^{\lambda_i\lambda_j} T_{D,\lambda_i} T_{D,\lambda_j}, \quad (13)$$

where $\Delta(\tilde\chi^0_i) = 1/[p_3^2 - m_i^2 + im_i\Gamma_i]$, m_i, Γ_i, and $\Delta(\tilde\chi^0_j) = 1/[p_4^2 - m_j^2 + im_j\Gamma_j]$, m_j, Γ_j denote the propagator, mass and width of $\tilde\chi^0_i$ and $\tilde\chi^0_j$, respectively. For these propagators we use the narrow width approximation.
The differential cross section in the laboratory system is then given by:

$$d\sigma = \frac{1}{8E_b^2}|T|^2(2\pi)^4\delta^4\left(p_1+p_2-\sum_i p_i\right) \times d\mathrm{lips}(p_3\dots p_{10}), \quad (14)$$

$E_b = \sqrt{s}/2$ denotes the beam energy and $d\mathrm{lips}(p_3,\dots,p_{10})$ is the Lorentz invariant phase space element.

The amplitude squared $|T|^2$ of the combined processes of production and decays, (13), is given by:

$$|T|^2 = 4|\Delta(\tilde\chi^0_i)|^2|\Delta(\tilde\chi^0_j)|^2\Big(PD(\tilde\chi^0_i)D(\tilde\chi^0_j) + \sum_{a=1}^{3}\Sigma_P^a(\tilde\chi^0_i)\Sigma_D^a(\tilde\chi^0_i)D(\tilde\chi^0_j) + \sum_{b=1}^{3}\Sigma_P^b(\tilde\chi^0_j)\Sigma_D^b(\tilde\chi^0_j)D(\tilde\chi^0_i) + \sum_{a,b=1}^{3}\Sigma_P^{ab}(\tilde\chi^0_i\tilde\chi^0_j)\Sigma_D^a(\tilde\chi^0_i)\Sigma_D^b(\tilde\chi^0_j)\Big). \quad (15)$$

Table 1. $c_L(\alpha\beta), c_R(\alpha\beta)$ for longitudinal beam polarization $P^3_-(P^3_+)$ of $e^-(e^+)$, α, β denote the exchanged particles, L_ℓ, R_ℓ are defined in (7). For unpolarized beams one has $P^3_- = P^3_+ = 0$.

$(\alpha\beta)$	$c_L(\alpha\beta)$	$c_R(\alpha\beta)$
(Z^0Z^0)	$L^2_\ell(1-P^3_-)(1+P^3_+)$	$R^2_\ell(1+P^3_-)(1-P^3_+)$
$(Z^0\tilde{e}_L)$	$L_\ell(1-P^3_-)(1+P^3_+)$	0
$(Z^0\tilde{e}_R)$	0	$R_\ell(1+P^3_-)(1-P^3_+)$
$(\tilde{e}_L\tilde{e}_L)$	$(1-P^3_-)(1+P^3_+)$	0
$(\tilde{e}_R\tilde{e}_R)$	0	$(1+P^3_-)(1-P^3_+)$

Here a, b=1, 2, 3 refer to the polarization vectors of $\tilde{\chi}^0_i$ and $\tilde{\chi}^0_j$ defined in (26)–(31) below. If one neglects all spin correlations between production and decay only the first term in (15) will contribute. The second and third term describe the spin correlations between the production and the decay processes $\tilde{\chi}^0_i \to \tilde{\chi}^0_k\ell^+\ell^-$ and $\tilde{\chi}^0_j \to \tilde{\chi}^0_\ell\ell^+\ell^-$, respectively, because $\Sigma^a_P(\tilde{\chi}^0_i)(\Sigma^b_P(\tilde{\chi}^0_j))$ as well as $\Sigma^a_D(\tilde{\chi}^0_i)$ $(\Sigma^b_D(\tilde{\chi}^0_j))$ depend on the polarization of the neutralino $\tilde{\chi}^0_i(\tilde{\chi}^0_j)$. Since $\Sigma^{ab}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ depends on the polarizations of both neutralinos the last term is due to spin-spin correlations between both decaying neutralinos (see Appendix (B.5)).

Owing to the Majorana character the spin correlations do not influence the energy distribution of the neutralino decay products and the opening angle distribution between the leptons from the decay of one of the neutralinos. Therefore, these distributions are given only by the first term $PD(\tilde{\chi}^0_i)D(\tilde{\chi}^0_j)$ in (15) [10,11].

We give the explicit expressions for P, $\Sigma^a_P(\tilde{\chi}^0_i)$, $\Sigma^b_P(\tilde{\chi}^0_j)$, $\Sigma^{ab}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ in Sect. 3 and for the quantities $D(\tilde{\chi}^0_i)$, $\Sigma^a_D(\tilde{\chi}^0_i)$, $D(\tilde{\chi}^0_j)$, $\Sigma^b_D(\tilde{\chi}^0_j)$ in Sect. 4.

3 Spin-density production matrix

In this section we give the analytical formulae for the quantities P, $\Sigma^a_P(\tilde{\chi}^0_i)$, $\Sigma^b_P(\tilde{\chi}^0_j)$, $\Sigma^{ab}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$, (15), for the production in the laboratory system.

It is useful to introduce the abbrevations $c_L(\alpha\beta)$, $c_R(\alpha\beta)$ as shown in Table 1. P^3_- (P^3_+) is the longitudinal beam polarization of $e^-(e^+)$, and $L_\ell(R_\ell)$ is defined in (7). The arguments α, β denote the exchanged particles. Generally $c_L(\alpha\beta)$ $(c_R(\alpha\beta))$ is large for $P^3_- < 0$, $P^3_+ > 0$ $(P^3_- > 0$, $P^3_+ < 0)$, and favours left(right) selectron exchange.

3.1 Neutralino polarization independent quantities

The expression P of (15) is independent of the neutralino polarization and reads:

$$P = P(ZZ) + P(Z\tilde{e}_L) + P(Z\tilde{e}_R) + P(\tilde{e}_L\tilde{e}_L) + P(\tilde{e}_R\tilde{e}_R), \quad (16)$$

with

$$P(ZZ) = 2\frac{g^4}{\cos^4\Theta_W}|\Delta^s(Z)|^2[c_R(ZZ)+c_L(ZZ)]E_b^2 \times\Big\{|O''^L_{ij}|^2E_iE_j - [(ReO''^L_{ij})^2-(ImO''^L_{ij})^2]\eta_i\eta_jm_im_j\Big\}, \quad (17)$$

$$P(Z\tilde{e}_L) = \frac{g^4}{\cos^2\Theta_W}c_L(Z\tilde{e}_L)E_b^2Re\Big\{\Delta^s(Z) \times\Big[-(\Delta^{t*}(\tilde{e}_L)f^{L*}_{\ell i}f^L_{\ell j}O''^{L*}_{ij} + \Delta^{u*}(\tilde{e}_L)f^L_{\ell i}f^{L*}_{\ell j}O''^L_{ij})\eta_i\eta_jm_im_j - (\Delta^{t*}(\tilde{e}_L)f^{L*}_{\ell i}f^L_{\ell j}O''^L_{ij} - \Delta^{u*}(\tilde{e}_L)f^L_{\ell i}f^{L*}_{\ell j}O''^{L*}_{ij})2E_bq\cos\Theta + (\Delta^{t*}(\tilde{e}_L)f^{L*}_{\ell i}f^L_{\ell j}O''^L_{ij} + \Delta^{u*}(\tilde{e}_L)f^L_{\ell i}f^{L*}_{\ell j}O''^{L*}_{ij}) \times(E_iE_j+q^2\cos^2\Theta)\Big]\Big\}, \quad (18)$$

$$P(\tilde{e}_L\tilde{e}_L) = \frac{g^4}{4}c_L(\tilde{e}_L\tilde{e}_L)E_b^2\Big\{|f^L_{\ell i}|^2|f^L_{\ell j}|^2 \times\Big[(|\Delta^t(\tilde{e}_L)|^2+|\Delta^u(\tilde{e}_L)|^2)(E_iE_j+q^2\cos^2\Theta) - (|\Delta^t(\tilde{e}_L)|^2-|\Delta^u(\tilde{e}_L)|^2)2E_bq\cos\Theta\Big] - Re\{(f^{L*}_{\ell i})^2(f^L_{\ell j})^2\Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L)\} \times 2\eta_i\eta_jm_im_j\Big\}. \quad (19)$$

$P(Z\tilde{e}_R), P(\tilde{e}_R\tilde{e}_R)$: To obtain these quantities one has to exchange in (18) and (19)

$$\Delta^t(\tilde{e}_L)\to\Delta^t(\tilde{e}_R), \quad \Delta^u(\tilde{e}_L)\to\Delta^u(\tilde{e}_R),$$
$$c_L(Z\tilde{e}_L)\to c_R(Z\tilde{e}_R), \quad c_L(\tilde{e}_L\tilde{e}_L)\to c_R(\tilde{e}_R\tilde{e}_R),$$
$$O''^L_{ij}\to O''^R_{ij}, \quad f^L_{\ell i}\to f^R_{\ell i}, \quad f^L_{\ell j}\to f^R_{\ell j}.$$

The propagators are defined as follows:

$$\Delta^s(Z) = \frac{i}{s-m_Z^2+im_Z\Gamma_Z},$$
$$\Delta^t(\tilde{e}_{L,R}) = \frac{i}{t-m^2_{\tilde{e}_{L,R}}+im_{\tilde{e}_{L,R}}\Gamma_{\tilde{e}_{L,R}}},$$
$$\Delta^u(\tilde{e}_{L,R}) = \frac{i}{u-m^2_{\tilde{e}_{L,R}}+im_{\tilde{e}_{L,R}}\Gamma_{\tilde{e}_{L,R}}}, \quad (20)$$

where $m_Z, \Gamma_Z, m_{\tilde{e}_L}, \Gamma_{\tilde{e}_L}, m_{\tilde{e}_R}, \Gamma_{\tilde{e}_R}$ denote the corresponding mass and width of the exchanged particle.

The angle Θ is the scattering angle between the incoming $e^-(p_1)$ beam and the outgoing neutralino $\tilde{\chi}^0_j(p_4)$, the azimuth can be chosen equal to zero. For our study of the whole process of production and subsequent decay it is convenient to choose a coordinate frame in the laboratory system, where the momenta are given by:

$$p_1 = E_b(1,-\sin\Theta,0,\cos\Theta), \quad (21)$$
$$p_2 = E_b(1,\sin\Theta,0,-\cos\Theta), \quad (22)$$
$$p_3 = (E_i,0,0,-q), \quad (23)$$
$$p_4 = (E_j,0,0,q), \quad (24)$$

with

$$E_i = \frac{s + m_i^2 - m_j^2}{2\sqrt{s}}, \quad E_j = \frac{s + m_j^2 - m_i^2}{2\sqrt{s}},$$
$$q = \frac{\lambda^{\frac{1}{2}}(4E_b^2, m_i^2, m_j^2)}{2\sqrt{s}}, \tag{25}$$

where m_i, m_j the masses of the neutralinos and λ the kinematical triangle function which is given by $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

3.2 Contributions of neutralino polarization

Now we give the terms of (15) which depend on the polarization states of the neutralinos. For the neutralino $\tilde{\chi}_i^0(\tilde{\chi}_j^0)$ with momentum $p_3(p_4)$ we introduce three spacelike polarization vectors $s_\mu^a(\tilde{\chi}_i^0)(s_\mu^b(\tilde{\chi}_j^0))$, (a, b=1, 2, 3), which together with $p_3^\mu/m(p_4^\mu/m)$ form an orthonormal set [19]. In the laboratory system, see (21)–(24), we choose the following set of polarization vectors:

$$s^1(\tilde{\chi}_i^0) = (0, -1, 0, 0), \tag{26}$$
$$s^2(\tilde{\chi}_i^0) = (0, 0, 1, 0), \tag{27}$$
$$s^3(\tilde{\chi}_i^0) = \frac{1}{m_i}(q, 0, 0, -E_i), \tag{28}$$
$$s^1(\tilde{\chi}_j^0) = (0, 1, 0, 0), \tag{29}$$
$$s^2(\tilde{\chi}_j^0) = (0, 0, 1, 0), \tag{30}$$
$$s^3(\tilde{\chi}_j^0) = \frac{1}{m_j}(q, 0, 0, E_j), \tag{31}$$

where s^3 denotes the longitudinal polarization, s^1 the transverse polarization in the scattering plane, and s^2 the transverse polarization perpendicular to the scattering plane.

3.2.1 Polarization of $\tilde{\chi}_i^0$

We give the expression for $\Sigma_P^a(\tilde{\chi}_i^0)$ of (15), where a =1, 2, 3 indicates the direction of the polarization vector $s^a(\tilde{\chi}_i^0)$, as given in (26)–(28). It can be decomposed as:

$$\Sigma_P^a(\tilde{\chi}_i^0) = \Sigma_P^a(\tilde{\chi}_i^0, ZZ) + \Sigma_P^a(\tilde{\chi}_i^0, Z\tilde{e}_L) + \Sigma_P^a(\tilde{\chi}_i^0, Z\tilde{e}_R) + \Sigma_P^a(\tilde{\chi}_i^0, \tilde{e}_L\tilde{e}_L) + \Sigma_P^a(\tilde{\chi}_i^0, \tilde{e}_R\tilde{e}_R). \tag{32}$$

1. The contributions of transverse polarization $s^1(\tilde{\chi}_i^0)$ *in the scattering plane* read:

$$\Sigma_P^1(\tilde{\chi}_i^0, ZZ) = 2\frac{g^4}{\cos^4\Theta_W}|\Delta^s(Z)|^2 E_b^2 \sin\Theta(c_R(ZZ) - c_L(ZZ))\Big[|O_{ij}^{''L}|^2\eta_i m_i E_j - [(ReO_{ij}^{''L})^2 - (ImO_{ij}^{''L})^2]\eta_j m_j E_i\Big], \tag{33}$$

$$\begin{aligned}\Sigma_P^1(\tilde{\chi}_i^0, Z\tilde{e}_L) = {} & \frac{g^4}{\cos^2\Theta_W}c_L(Z\tilde{e}_L)E_b^2\sin\Theta \\ & \times\Big[-Re\big\{\Delta^s(Z)\big[f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L*}\Delta^{u*}(\tilde{e}_L) \\ & + f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L}\Delta^{t*}(\tilde{e}_L)\big]\eta_i m_i E_j\big\} \\ & + Re\big\{\Delta^s(Z)\big[f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L}\Delta^{u*}(\tilde{e}_L) \\ & + f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L*}\Delta^{t*}(\tilde{e}_L)\big]\eta_j m_j E_i\big\} \\ & - Re\big\{\Delta^s(Z)\big[f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L*}\Delta^{u*}(\tilde{e}_L) \\ & - f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L}\Delta^{t*}(\tilde{e}_L)\big] \\ & \times \eta_i m_i q\cos\Theta\big\}\Big],\end{aligned} \tag{34}$$

$$\begin{aligned}\Sigma_P^1(\tilde{\chi}_i^0, \tilde{e}_L\tilde{e}_L) = {} & -\frac{g^4}{4}c_L(\tilde{e}_L\tilde{e}_L)E_b^2\sin\Theta\Big\{|f_{\ell i}^L|^2|f_{\ell j}^L|^2 \\ & \times\Big[(|\Delta^t(\tilde{e}_L)|^2 + |\Delta^u(\tilde{e}_L)|^2)\eta_i m_i E_j \\ & - (|\Delta^t(\tilde{e}_L)|^2 - |\Delta^u(\tilde{e}_L)|^2)\eta_i m_i q\cos\Theta\Big] \\ & - 2Re\{(f_{\ell i}^{L*})^2(f_{\ell j}^L)^2\Delta^u(\tilde{e}_L) \\ & \times \Delta^{t*}(\tilde{e}_L)\}\eta_j m_j E_i\Big\}.\end{aligned} \tag{35}$$

$\Sigma_P^1(\tilde{\chi}_i^0, Z\tilde{e}_R), \Sigma_P^1(\tilde{\chi}_i^0, \tilde{e}_R\tilde{e}_R)$: To obtain these quantities one has to exchange in (34) and (35)

$$\Delta^t(\tilde{e}_L) \to \Delta^t(\tilde{e}_R), \quad \Delta^u(\tilde{e}_L) \to \Delta^u(\tilde{e}_R),$$
$$c_L(Z\tilde{e}_L) \to c_R(Z\tilde{e}_R), \quad c_L(\tilde{e}_L\tilde{e}_L) \to c_R(\tilde{e}_R\tilde{e}_R),$$
$$O_{ij}^{''L} \to O_{ij}^{''R}, \quad f_{\ell i}^L \to f_{\ell i}^R, \quad f_{\ell j}^L \to f_{\ell j}^R,$$

and to change the overall sign of the right hand side of (34), (35).

2. The contributions of longitudinal polarization $s^3(\tilde{\chi}_i^0)$ read:

$$\begin{aligned}&\Sigma_P^3(\tilde{\chi}_i^0, ZZ) \\ &= \eta_i\frac{2g^4}{\cos^4\Theta_W}|\Delta^s(Z)|^2(c_L(ZZ) - c_R(ZZ))E_b^2\cos\Theta \\ &\times\Big[|O_{ij}^{''L}|^2(E_iE_j + q^2) \\ &-[(ReO_{ij}^{''L})^2 - (ImO_{ij}^{''L*})^2]\eta_i\eta_j m_i m_j\Big],\end{aligned} \tag{36}$$

$$\begin{aligned}&\Sigma_P^3(\tilde{\chi}_i^0, Z\tilde{e}_L) \\ &= \eta_i\frac{g^4}{\cos^2\Theta_W}c_L(Z\tilde{e}_L)E_b^2 \\ &\times\Big[Re\Big\{\Delta^s(Z)[f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L*}\Delta^{u*}(\tilde{e}_L) \\ &-f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L}\Delta^{t*}(\tilde{e}_L)]E_j q\Big\} \\ &+Re\Big\{\Delta^s(Z)[f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L*}\Delta^{u*}(\tilde{e}_L) \\ &+f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L}\Delta^{t*}(\tilde{e}_L)](E_iE_j + q^2)\cos\Theta\Big\} \\ &+Re\Big\{\Delta^s(Z)[f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L*}\Delta^{u*}(\tilde{e}_L) \\ &-f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L}\Delta^{t*}(\tilde{e}_L)]E_i q\cos^2\Theta\Big\} \\ &-Re\Big\{\Delta^s(Z)[f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L}\Delta^{u*}(\tilde{e}_L) \\ &+f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L*}\Delta^{t*}(\tilde{e}_L)]\eta_i\eta_j m_i m_j\cos\Theta\Big\}\Big],\end{aligned} \tag{37}$$

$$\Sigma^3_P(\tilde\chi^0_i, \tilde e_L \tilde e_L)$$
$$= \eta_i \frac{g^4}{4} c_L(\tilde e_L \tilde e_L) E_b^2 \Big[|f^L_{\ell i}|^2 |f^L_{\ell j}|^2$$
$$\times \Big\{ [|\Delta^u(\tilde e_L)|^2 - |\Delta^t(\tilde e_L)|^2] E_j q + [|\Delta^u(\tilde e_L)|^2$$
$$- |\Delta^t(\tilde e_L)|^2] q E_i \cos^2\Theta$$
$$+ [|\Delta^t(\tilde e_L)|^2 + |\Delta^u(\tilde e_L)|^2](E_i E_j + q^2)\cos\Theta \Big\}$$
$$-2Re\{(f^{L*}_{\ell i})^2 (f^L_{\ell j})^2$$
$$\times \Delta^u(\tilde e_L) \Delta^{t*}(\tilde e_L)\} \eta_i \eta_j m_i m_j \cos\Theta \Big]. \quad (38)$$

$\Sigma^3_P(\tilde\chi^0_i, Z\tilde e_R), \Sigma^3_P(\tilde\chi^0_i, \tilde e_R \tilde e_R)$: To obtain these quantities one has to exchange in (37) and (38)

$$\Delta^t(\tilde e_L) \to \Delta^t(\tilde e_R), \quad \Delta^u(\tilde e_L) \to \Delta^u(\tilde e_R),$$
$$c_L(Z\tilde e_L) \to c_R(Z\tilde e_R), \quad c_L(\tilde e_L \tilde e_L) \to c_R(\tilde e_R \tilde e_R),$$
$$O''^L_{ij} \to O''^R_{ij}, \quad f^L_{\ell i} \to f^R_{\ell i}, \quad f^L_{\ell j} \to f^R_{\ell j}.$$

and to change the overall sign of (37), (38).

3. The contributions of the polarization $s^2(\tilde\chi^0_i)$ *perpendicular to the scattering plane* are:

$$\Sigma^2_P(\tilde\chi^0_i, ZZ)$$
$$= -4 \left(\frac{g^2}{\cos^2\Theta_W} \right)^2 |\Delta^s(Z)|^2 (c_R(ZZ) - c_L(ZZ))$$
$$\times m_j q E_b^2 \sin\Theta Re(O''^L_{ij}) Im(O''^L_{ij}), \quad (39)$$
$$\Sigma^2_P(\tilde\chi^0_i, Z\tilde e_L)$$
$$= \frac{g^4}{\cos^2\Theta_W} c_L(Z\tilde e_L) \eta_j m_j E_b^2 q \sin\Theta$$
$$\times Im \Big\{ \Delta^s(Z) \big[f^L_{\ell i} f^{L*}_{\ell j} O''^L_{ij} \Delta^{u*}(\tilde e_L)$$
$$- f^{L*}_{\ell i} f^L_{\ell j} O''^{L*}_{ij} \Delta^{t*}(\tilde e_L) \big] \Big\}, \quad (40)$$
$$\Sigma^2_P(\tilde\chi^0_i, \tilde e_L \tilde e_L)$$
$$= -\frac{g^4}{2} c_L(\tilde e_L \tilde e_L) \eta_j m_j E_b^2 q \sin\Theta$$
$$\times Im \Big\{ (f^{L*}_{\ell i})^2 (f^L_{\ell j})^2 \Delta^u(\tilde e_L) \Delta^{t*}(\tilde e_L) \Big\}. \quad (41)$$

$\Sigma^2_P(\tilde\chi^0_i, Z\tilde e_R), \Sigma^2_P(\tilde\chi^0_i, \tilde e_R \tilde e_R)$: To obtain these quantities one has to exchange in (40) and (41)

$$\Delta^t(\tilde e_L) \to \Delta^t(\tilde e_R), \quad \Delta^u(\tilde e_L) \to \Delta^u(\tilde e_R),$$
$$c_L(Z\tilde e_L) \to c_R(Z\tilde e_R), \quad c_L(\tilde e_L \tilde e_L) \to c_R(\tilde e_R \tilde e_R),$$
$$O''^L_{ij} \to O''^R_{ij}, \quad f^L_{\ell i} \to f^R_{\ell i}, \quad f^L_{\ell j} \to f^R_{\ell j}.$$

Contrary to the case of $s^1(\tilde\chi^0_i)$ and $s^3(\tilde\chi^0_i)$ the sign of the contributions $\Sigma^2_P(\tilde\chi^0_i)$ does not change when going from $\tilde e_L$ exchange to $\tilde e_R$ exchange.

3.2.2 Polarization of $\tilde\chi^0_j$

We give the quantities $\Sigma^b_P(\tilde\chi^0_j)$ of (15) which contain only the polarization vector $s^b(\tilde\chi^0_j)$ with b =1, 2, 3, (29)–(31):

$$\Sigma^b_P(\tilde\chi^0_j) = \Sigma^b_P(\tilde\chi^0_j, ZZ) + \Sigma^b_P(\tilde\chi^0_j, Z\tilde e_L) + \Sigma^b_P(\tilde\chi^0_j, Z\tilde e_R)$$
$$+ \Sigma^b_P(\tilde\chi^0_j, \tilde e_L \tilde e_L) + \Sigma^b_P(\tilde\chi^0_j, \tilde e_R \tilde e_R). \quad (42)$$

1. $\Sigma^1_P(\tilde\chi^0_j)$, $\Sigma^3_P(\tilde\chi^0_j)$: The expressions for $\Sigma^1_P(\tilde\chi^0_j)$, $\Sigma^3_P(\tilde\chi^0_j)$ are obtained from those of $\Sigma^1_P(\tilde\chi^0_i)$, $\Sigma^3_P(\tilde\chi^0_i)$, (33)–(38), by exchanging

$$m_i \to m_j, \quad \eta_i \to \eta_j, \quad E_i \to E_j, \quad (43)$$

and by changing the overall sign of these expressions, for example,

$$\Sigma^1_P(\tilde\chi^0_j, ZZ)$$
$$= -2 \frac{g^4}{\cos^4\Theta_W} |\Delta(Z)|^2 E_b^2 \sin\Theta (c_R(ZZ) - c_L(ZZ))$$
$$\times \Big[|O''^L_{ij}|^2 \eta_j m_j E_i$$
$$- [(ReO''^L_{ij})^2 - (ImO''^L_{ij})^2] \eta_i m_i E_j \Big]. \quad (44)$$

2. $\Sigma^2_P(\tilde\chi^0_j)$: The expressions for $\Sigma^2_P(\tilde\chi^0_j)$ are obtained from those for $\Sigma^2_P(\tilde\chi^0_i)$, (39)–(41) by exchanging

$$\eta_i \to \eta_j, \quad m_i \to m_j, \quad E_i \to E_j \quad (45)$$

(without changing the overall sign).

Note that

- the transverse polarizations $\Sigma^1_P(\tilde\chi^0_i)$, $\Sigma^2_P(\tilde\chi^0_i)$, $\Sigma^1_P(\tilde\chi^0_j)$, $\Sigma^2_P(\tilde\chi^0_j)$ of the neutralinos vanish in forward and backward direction;
- at threshold the tranverse polarizations $\Sigma^2_P(\tilde\chi^0_i)$ and $\Sigma^2_P(\tilde\chi^0_j)$ perpendicular to the production plane vanish proportional to the momentum of the neutralinos.

3.2.3 Spin-spin correlations

We give the expressions for $\Sigma^{ab}_P(\tilde\chi^0_i \tilde\chi^0_j)$ of (15), where a, b =1, 2, 3 indicate the directions of the polarization vectors $s^a(\tilde\chi^0_i)$ and $s^b(\tilde\chi^0_j)$ as given in (26)–(31). They can be decomposed as:

$$\Sigma^{ab}_P(\tilde\chi^0_i \tilde\chi^0_j)$$
$$= \Sigma^{ab}_P(\tilde\chi^0_i \tilde\chi^0_j, ZZ) + \Sigma^{ab}_P(\tilde\chi^0_i \tilde\chi^0_j, Z\tilde e_L)$$
$$+ \Sigma^{ab}_P(\tilde\chi^0_i \tilde\chi^0_j, Z\tilde e_R) + \Sigma^{ab}_P(\tilde\chi^0_i \tilde\chi^0_j, \tilde e_L \tilde e_L)$$
$$+ \Sigma^{ab}_P(\tilde\chi^0_i \tilde\chi^0_j, \tilde e_R \tilde e_R), \quad \text{with} \quad a,b = 1,2,3. \quad (46)$$

1. The contributions of $s^1(\tilde\chi^0_i)$ and $s^1(\tilde\chi^0_j)$ are:

$$\Sigma^{11}_P(\tilde\chi^0_i \tilde\chi^0_j, ZZ)$$
$$= 2 \frac{g^4}{\cos^4\Theta_W} |\Delta^s(Z)|^2 (c_R(ZZ) + c_L(ZZ)) E_b^2 \sin^2\Theta$$
$$\times \{ [(ReO''^L_{ij})^2 - (ImO''^L_{ij})^2] E_i E_j$$
$$-2 |O''^L_{ij}|^2 \eta_i \eta_j m_i m_j \}, \quad (47)$$
$$\Sigma^{11}_P(\tilde\chi^0_i \tilde\chi^0_j, Z\tilde e_L)$$
$$= \frac{g^4}{\cos^2\Theta_W} c_L(Z\tilde e_L) E_b^2 \sin^2\Theta$$

$$\times Re\Big\{[f^L_{\ell i} f^{L*}_{\ell j} \Delta^s(Z)\Delta^{u*}(\tilde{e}_L) O''^L_{ij}$$
$$+ f^{L*}_{\ell i} f^L_{\ell j} \Delta^s(Z)\Delta^{t*}(\tilde{e}_L) O''^{L*}_{ij}] E_i E_j$$
$$-[f^L_{\ell i} f^{L*}_{\ell j} \Delta^s(Z)\Delta^{u*}(\tilde{e}_L) O''^{L*}_{ij}$$
$$+ f^{L*}_{\ell i} f^L_{\ell j} \Delta^s(Z)\Delta^{t*}(\tilde{e}_L) O''^L_{ij}] \eta_i\eta_j m_i m_j\Big\}, \quad (48)$$

$$\Sigma^{11}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, \tilde{e}_L\tilde{e}_L)$$
$$= -\frac{g^4}{4} c_L(\tilde{e}_L\tilde{e}_L) E_b^2 \sin^2\Theta$$
$$\times\Big[|f^L_{\ell i}|^2 |f^L_{\ell j}|^2 (|\Delta^t(\tilde{e}_L)|^2 + |\Delta^u(\tilde{e}_L)|^2)\eta_i\eta_j m_i m_j$$
$$+2Re\big\{(f^{L*}_{\ell i})^2 (f^L_{\ell j})^2 \Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L)\big\} E_i E_j\Big]. \quad (49)$$

$\Sigma^{11}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, Z\tilde{e}_R), \Sigma^{11}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, \tilde{e}_R\tilde{e}_R)$: To obtain these quantities one has to exchange in (48) and (49)

$$\Delta^t(\tilde{e}_L) \to \Delta^t(\tilde{e}_R), \quad \Delta^u(\tilde{e}_L) \to \Delta^u(\tilde{e}_R),$$
$$c_L(Z\tilde{e}_L) \to c_R(Z\tilde{e}_R), \quad c_L(\tilde{e}_L\tilde{e}_L) \to c_R(\tilde{e}_R\tilde{e}_R),$$
$$O''^L_{ij} \to O''^R_{ij}, \quad f^L_{\ell i} \to f^R_{\ell i}, \quad f^L_{\ell j} \to f^R_{\ell j}.$$

2. The contributions of $s^2(\tilde{\chi}^0_i)$ and $s^2(\tilde{\chi}^0_j)$ are:

$$\Sigma^{22}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, ZZ)$$
$$= 2\frac{g^4}{\cos^4\Theta_W} |\Delta^s(Z)|^2 (c_R(ZZ) + c_L(ZZ)) E_b^2 q^2$$
$$\times \sin^2\Theta\{(ReO''^L_{ij})^2 - (ImO''^L_{ij})^2\}, \quad (50)$$

$$\Sigma^{22}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, Z\tilde{e}_L)$$
$$= \frac{g^4}{\cos^2\Theta_W} c_L(Z\tilde{e}_L) E_b^2 q^2 \sin^2\Theta Re\Big\{\Delta^s(Z)$$
$$\times[f^L_{\ell i} f^{L*}_{\ell j} \Delta^{u*}(\tilde{e}_L) O''^L_{ij} + f^{L*}_{\ell i} f^L_{\ell j} \Delta^{t*}(\tilde{e}_L) O''^{L*}_{ij}]\Big\}, \quad (51)$$

$$\Sigma^{22}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, \tilde{e}_L\tilde{e}_L)$$
$$= -\frac{g^4}{2} c_L(\tilde{e}_L\tilde{e}_L) E_b^2 q^2 \sin^2\Theta$$
$$\times Re\Big\{(f^{L*}_{\ell i})^2 (f^L_{\ell j})^2 \Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L)\Big\}. \quad (52)$$

$\Sigma^{22}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, Z\tilde{e}_R), \Sigma^{22}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, \tilde{e}_R\tilde{e}_R)$: To obtain these quantities one has to exchange in (51) and (52)

$$\Delta^t(\tilde{e}_L) \to \Delta^t(\tilde{e}_R), \quad \Delta^u(\tilde{e}_L) \to \Delta^u(\tilde{e}_R),$$
$$c_L(Z\tilde{e}_L) \to c_R(Z\tilde{e}_R), \quad c_L(\tilde{e}_L\tilde{e}_L) \to c_R(\tilde{e}_R\tilde{e}_R),$$
$$O''^L_{ij} \to O''^R_{ij}, \quad f^L_{\ell i} \to f^R_{\ell i}, \quad f^L_{\ell j} \to f^R_{\ell j}.$$

3. The contributions of $s^3(\tilde{\chi}^0_i)$ and $s^3(\tilde{\chi}^0_j)$ are:

$$\Sigma^{33}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, ZZ)$$
$$= \eta_i\eta_j \frac{2g^4}{\cos^4\Theta_W} |\Delta^s(Z)|^2 (c_R(ZZ) + c_L(ZZ)) E_b^2$$
$$\times\Big[((ReO''^L_{ij})^2 - (ImO''^L_{ij})^2)\eta_i\eta_j m_i m_j \cos^2\Theta$$
$$-|O''^L_{ij}|^2 [q^2 + E_i E_j \cos^2\Theta]\Big], \quad (53)$$

$$\Sigma^{33}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, Z\tilde{e}_L)$$
$$= \eta_i\eta_j \frac{g^4}{\cos^2\Theta_W} c_L(Z\tilde{e}_L) E_b^2$$
$$\times\Big[Re\big\{\Delta^s(Z)[f^L_{\ell i} f^{L*}_{\ell j} \Delta^{u*}(\tilde{e}_L) O''^L_{ij}$$
$$+ f^{L*}_{\ell i} f^L_{\ell j} \Delta^{t*}(\tilde{e}_L) O''^{L*}_{ij}]\big\}\eta_i\eta_j m_i m_j \cos^2\Theta$$
$$-Re\big\{\Delta^s(Z)[f^L_{\ell i} f^{L*}_{\ell j} \Delta^{u*}(\tilde{e}_L) O''^{L*}_{ij}$$
$$+ f^{L*}_{\ell i} f^L_{\ell j} \Delta^{t*}(\tilde{e}_L) O''^L_{ij}]\big\}[q^2 + E_i E_j \cos^2\Theta]$$
$$-Re\big\{\Delta^s(Z)[f^L_{\ell i} f^{L*}_{\ell j} \Delta^{u*}(\tilde{e}_L) O''^{L*}_{ij}$$
$$- f^{L*}_{\ell i} f^L_{\ell j} \Delta^{t*}(\tilde{e}_L) O''^L_{ij}]\big\} 2E_b q \cos\Theta\Big], \quad (54)$$

$$\Sigma^{33}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, \tilde{e}_L\tilde{e}_L)$$
$$= \eta_i\eta_j \frac{g^4}{4} c_L(\tilde{e}_L\tilde{e}_L) E_b^2 \Big[|f^L_{\ell i}|^2 |f^L_{\ell j}|^2$$
$$\times\Big(-(|\Delta^t(\tilde{e}_L)|^2 + |\Delta^u(\tilde{e}_L)|^2)[q^2 + E_i E_j \cos^2\Theta]$$
$$+(|\Delta^t(\tilde{e}_L)|^2 - |\Delta^u(\tilde{e}_L)|^2) 2E_b q \cos\Theta\Big)$$
$$-2Re\big\{(f^{L*}_{\ell i})^2 (f^L_{\ell j})^2 \Delta^u(\tilde{e}_L)$$
$$\times\Delta^{t*}(\tilde{e}_L)\big\}\eta_i\eta_j m_i m_j \cos^2\Theta\Big]. \quad (55)$$

$\Sigma^{33}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, Z\tilde{e}_R), \Sigma^{33}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, \tilde{e}_R\tilde{e}_R)$: To obtain these quantities one has to exchange in (54) and (55)

$$\Delta^t(\tilde{e}_L) \to \Delta^t(\tilde{e}_R), \quad \Delta^u(\tilde{e}_L) \to \Delta^u(\tilde{e}_R),$$
$$c_L(Z\tilde{e}_L) \to c_R(Z\tilde{e}_R), \quad c_L(\tilde{e}_L\tilde{e}_L) \to c_R(\tilde{e}_R\tilde{e}_R),$$
$$O''^L_{ij} \to O''^R_{ij}, \quad f^L_{\ell i} \to f^R_{\ell i}, \quad f^L_{\ell j} \to f^R_{\ell j}.$$

4. The contributions of $s^1(\tilde{\chi}^0_i)$ and $s^3(\tilde{\chi}^0_j)$ are:

$$\Sigma^{13}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, ZZ)$$
$$= \eta_j \frac{2g^4}{\cos^4\Theta_W} |\Delta^s(Z)|^2 (c_R(ZZ) + c_L(ZZ)) E_b^2$$
$$\times \sin\Theta\cos\Theta\Big[-((ReO''^L_{ij})^2 - (ImO''^L_{ij})^2) E_i\eta_j m_j$$
$$+|O''^L_{ij}|^2 \eta_i m_i E_j\Big], \quad (56)$$

$$\Sigma^{13}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, Z\tilde{e}_L)$$
$$= \eta_j \frac{g^4}{\cos^2\Theta_W} c_L(Z\tilde{e}_L) E_b^2 \sin\Theta\Big[-Re\big\{\Delta^s(Z)$$
$$\times[f^L_{\ell i} f^{L*}_{\ell j} \Delta^{u*}(\tilde{e}_L) O''^L_{ij} + f^{L*}_{\ell i} f^L_{\ell j} \Delta^{t*}(\tilde{e}_L) O''^{L*}_{ij}]\big\}$$
$$\times E_i\eta_j m_j \cos\Theta + Re\big\{\Delta^s(Z)[f^L_{\ell i} f^{L*}_{\ell j} \Delta^{u*}(\tilde{e}_L) O''^{L*}_{ij}$$
$$+ f^{L*}_{\ell i} f^L_{\ell j} \Delta^{t*}(\tilde{e}_L) O''^L_{ij}]\big\}\eta_i m_i E_j \cos\Theta$$
$$+Re\big\{\Delta^s(Z)[f^L_{\ell i} f^{L*}_{\ell j} \Delta^{u*}(\tilde{e}_L) O''^{L*}_{ij}$$
$$- f^{L*}_{\ell i} f^L_{\ell j} \Delta^{t*}(\tilde{e}_L) O''^L_{ij}]\big\}\eta_i m_i q\Big], \quad (57)$$

$$\Sigma^{13}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j, \tilde{e}_L\tilde{e}_L)$$
$$= \eta_j \frac{g^4}{4} c_L(\tilde{e}_L\tilde{e}_L) E_b^2 \sin\Theta$$

$$\times\Big[|f^L_{\ell i}|^2|f^L_{\ell j}|^2\big\{[|\Delta^u(\tilde{e}_L)|^2-|\Delta^t(\tilde{e}_L)|^2]\eta_i m_i q$$
$$+[|\Delta^t(\tilde{e}_L)|^2+|\Delta^u(\tilde{e}_L)|^2]\eta_i m_i E_j\cos\Theta\big\}$$
$$+2Re\big\{(f^{L*}_{\ell i})^2(f^L_{\ell j})^2\Delta^u(\tilde{e}_L)$$
$$\times\Delta^{t*}(\tilde{e}_L)\big\}E_i\eta_j m_j\cos\Theta\Big]. \quad (58)$$

$\Sigma^{13}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,Z\tilde{e}_R),\Sigma^{13}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,\tilde{e}_R\tilde{e}_R)$: To obtain these quantities one has to exchange in (57) and (58)

$$\Delta^t(\tilde{e}_L)\to\Delta^t(\tilde{e}_R),\quad \Delta^u(\tilde{e}_L)\to\Delta^u(\tilde{e}_R)$$
$$c_L(Z\tilde{e}_L)\to c_R(Z\tilde{e}_R),\quad c_L(\tilde{e}_L\tilde{e}_L)\to c_R(\tilde{e}_R\tilde{e}_R),$$
$$O''^L_{ij}\to O''^R_{ij},\quad f^L_{\ell i}\to f^R_{\ell i},\quad f^L_{\ell j}\to f^R_{\ell j}.$$

5. The contributions of $s^3(\tilde{\chi}^0_i)$ and $s^1(\tilde{\chi}^0_j)$ are:
The expressions for $\Sigma^{31}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ are obtained by exchanging

$$\eta_i\leftrightarrow\eta_j,\quad m_i\leftrightarrow m_j,\quad E_i\leftrightarrow E_j \quad (59)$$

in (56)–(58) and also in the corresponding contributions from $\tilde{e}_R$ exchange.
6. The contributions of $s^1(\tilde{\chi}^0_i)$ and $s^2(\tilde{\chi}^0_j)$ are:

$$\Sigma^{12}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,ZZ)$$
$$=2\frac{g^4}{\cos^4\Theta_W}|\Delta^s(Z)|^2(c_R(ZZ)+c_L(ZZ))E_b^2E_i q$$
$$\times\sin^2\Theta Re(O''^L_{ij})Im(O''^L_{ij}), \quad (60)$$
$$\Sigma^{12}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,Z\tilde{e}_L)$$
$$=\frac{g^4}{\cos^2\Theta_W}c_L(Z\tilde{e}_L)E_b^2E_i q\sin^2\Theta Im\Big\{\Delta^s(Z)[f^L_{\ell i}f^{L*}_{\ell j}$$
$$\times\Delta^{u*}(\tilde{e}_L)O''^L_{ij}+f^{L*}_{\ell i}f^L_{\ell j}\Delta^{t*}(\tilde{e}_L)O''^{L*}_{ij}]\Big\}, \quad (61)$$
$$\Sigma^{12}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,\tilde{e}_L\tilde{e}_L)$$
$$=\frac{g^4}{2}c_L(\tilde{e}_L\tilde{e}_L)E_b^2E_i q\sin^2\Theta$$
$$\times Im\Big\{(f^{L*}_{\ell i})^2(f^L_{\ell j})^2\Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L)\Big\}. \quad (62)$$

$\Sigma^{12}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,Z\tilde{e}_R),\Sigma^{12}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,\tilde{e}_R\tilde{e}_R)$: To obtain these quantities one has to exchange in (61) and (62)

$$\Delta^t(\tilde{e}_L)\to\Delta^t(\tilde{e}_R),\quad \Delta^u(\tilde{e}_L)\to\Delta^u(\tilde{e}_R),$$
$$c_L(Z\tilde{e}_L)\to c_R(Z\tilde{e}_R),\quad c_L(\tilde{e}_L\tilde{e}_L)\to c_R(\tilde{e}_R\tilde{e}_R),$$
$$O''^L_{ij}\to O''^R_{ij},\quad f^L_{\ell i}\to f^R_{\ell i},\quad f^L_{\ell j}\to f^R_{\ell j},$$

and to change the overall sign of (61), (62).
7. The contributions of $s^2(\tilde{\chi}^0_i)$ and $s^1(\tilde{\chi}^0_j)$ are:
The expressions for $\Sigma^{21}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ are obtained by exchanging

$$\eta_i\leftrightarrow\eta_j,\quad m_i\leftrightarrow m_j,\quad E_i\leftrightarrow E_j \quad (63)$$

in (60)–(62) and in the corresponding contributions from $\tilde{e}_R$ exchange. In addition, one also has to change the overall sign.
8. The contributions of $s^2(\tilde{\chi}^0_i)$ and $s^3(\tilde{\chi}^0_j)$ are:

$$\Sigma^{23}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,ZZ)$$
$$=\frac{2g^4}{\cos^4\Theta_W}|\Delta^s(Z)|^2[c_R(ZZ)+c_L(ZZ)]$$
$$\times m_jE_b^2q\sin\Theta\cos\Theta Re(O''^L_{ij})Im(O''^L_{ij}), \quad (64)$$
$$\Sigma^{23}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,Z\tilde{e}_L)$$
$$=\frac{g^4}{\cos^2\Theta_W}c_L(Z\tilde{e}_L)E_b^2m_jq\sin\Theta\cos\Theta Im\Big\{\Delta^s(Z)$$
$$\times[f^L_{\ell i}f^{L*}_{\ell j}\Delta^{u*}(\tilde{e}_L)O''^L_{ij}+f^{L*}_{\ell i}f^L_{\ell j}\Delta^{t*}(\tilde{e}_L)O''^{L*}_{ij}]\Big\}, \quad (65)$$
$$\Sigma^{23}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,\tilde{e}_L\tilde{e}_L)$$
$$=\frac{g^4}{2}c_L(\tilde{e}_L\tilde{e}_L)E_b^2m_jq\sin\Theta\cos\Theta$$
$$\times Im\Big\{(f^{L*}_{\ell i})^2(f^L_{\ell j})^2\Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L)\Big\}. \quad (66)$$

$\Sigma^{23}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,Z\tilde{e}_R),\Sigma^{23}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j,\tilde{e}_R\tilde{e}_R)$: To obtain these quantities one has to exchange in (65) and (66)

$$\Delta^t(\tilde{e}_L)\to\Delta^t(\tilde{e}_R),\quad \Delta^u(\tilde{e}_L)\to\Delta^u(\tilde{e}_R),$$
$$c_L(Z\tilde{e}_L)\to c_R(Z\tilde{e}_R),\quad c_L(\tilde{e}_L\tilde{e}_L)\to c_R(\tilde{e}_R\tilde{e}_R),$$
$$O''^L_{ij}\to O''^R_{ij},\quad f^L_{\ell i}\to f^R_{\ell i},\quad f^L_{\ell j}\to f^R_{\ell j},$$

and to change the overall sign of (65), (66).
9. The contributions of $s^3(\tilde{\chi}^0_i)$ and $s^2(\tilde{\chi}^0_j)$ are:
The expressions for $\Sigma^{32}_P(\tilde{\chi}^0_i\tilde{\chi}^0_j)$ are obtained by exchanging

$$m_i\leftrightarrow m_j,\quad E_i\leftrightarrow E_j \quad (67)$$

in (64)–(66) and in the corresponding contributions from $\tilde{e}_R$ exchange. In addition, one also has to change the overall sign.

Note that

– all contributions of transverse polarizations $s^1(\tilde{\chi}^0_i)$, $s^2(\tilde{\chi}^0_i)$, $s^1(\tilde{\chi}^0_j)$, $s^2(\tilde{\chi}^0_j)$ vanish in forward and backward direction;
– at threshold all spin-spin terms of transverse polarizations $s^2(\tilde{\chi}^0_i)$, $s^2(\tilde{\chi}^0_j)$ vanish proportional to the momentum of the neutralinos.

4 Decay matrix

In the following we give the analytical formulae for the decay matrices $D(\tilde{\chi}^0_i)$, $\Sigma^a_D(\tilde{\chi}^0_i)$ for the decay $\tilde{\chi}^0_i(p_3)\to\tilde{\chi}^0_k(p_5)+\ell^+(p_6)+\ell^-(p_7)$, and $D(\tilde{\chi}^0_j)$, $\Sigma^b_D(\tilde{\chi}^0_j)$ for the decay $\tilde{\chi}^0_j(p_4)\to\tilde{\chi}^0_l(p_8)+\ell^+(p_9)+\ell^-(p_{10})$. We present them in covariant form. They have to be inserted in (15) to obtain the amplitude squared for the combined process of neutralino production and decay.

4.1 Neutralino polarization independent quantities

The expression $D(\tilde{\chi}^0_i)$ of (15) which is independent of the polarization vector $s^a(\tilde{\chi}^0_i)$ has the following decomposition:

$$D(\tilde{\chi}_i^0) = D(\tilde{\chi}_i^0, ZZ) + D(\tilde{\chi}_i^0, Z\tilde{\ell}_L) + D(\tilde{\chi}_i^0, Z\tilde{\ell}_R) + D(\tilde{\chi}_i^0, \tilde{\ell}_L\tilde{\ell}_L) + D(\tilde{\chi}_i^0, \tilde{\ell}_R\tilde{\ell}_R). \quad (68)$$

The analytical expressions for $D(\tilde{\chi}_i^0)$, (68), read:

$$D(\tilde{\chi}_i^0, ZZ) = 8\frac{g^4}{\cos^4\Theta_W}|\Delta^{s_i}(Z)|^2(L_\ell^2 + R_\ell^2)\Big[|O_{ki}^{''L}|^2(g_1 + g_2) + [(ReO_{ki}^{''L})^2 - (ImO_{ki}^{''L})^2]g_3\Big], \quad (69)$$

$$D(\tilde{\chi}_i^0, Z\tilde{\ell}_L) = 4\frac{g^4}{\cos^2\Theta_W}L_\ell Re\Big\{\Delta^{s_i}(Z)\big[f_{\ell i}^L f_{\ell k}^{L*}\Delta^{t_i*}(\tilde{\ell}_L)(2O_{ki}^{''L}g_1 + O_{ki}^{''L*}g_3) + f_{\ell i}^{L*}f_{\ell k}^L\Delta^{u_i*}(\tilde{\ell}_L)(2O_{ki}^{''L*}g_2 + O_{ki}^{''L}g_3)\big]\Big\}, \quad (70)$$

$$D(\tilde{\chi}_i^0, \tilde{\ell}_L\tilde{\ell}_L) = 2g^4\Big[|f_{\ell i}^L|^2|f_{\ell k}^L|^2\big(|\Delta^{t_i}(\tilde{\ell}_L)|^2 g_1 + |\Delta^{u_i}(\tilde{\ell}_L)|^2 g_2\big) + Re\big\{(f_{\ell i}^{L*})^2(f_{\ell k}^L)^2\Delta^{t_i}(\tilde{\ell}_L)\Delta^{u_i*}(\tilde{\ell}_L)\big\}g_3\Big], \quad (71)$$

where we have introduced the following combinations of scalar products:

$$g_1 = (p_5p_7)(p_3p_6), \quad (72)$$
$$g_2 = (p_5p_6)(p_3p_7), \quad (73)$$
$$g_3 = \eta_i\eta_k m_i m_k(p_6p_7). \quad (74)$$

The propagators are denoted by $\Delta^{s_i}(Z)$, $\Delta^{t_i}(\tilde{\ell}_{L,R})$, $\Delta^{u_i}(\tilde{\ell}_{L,R})$ and are defined analogously to (20), with s_i, t_i, u_i as defined after (12).

$D(\tilde{\chi}_i^0, Z\tilde{\ell}_R), D(\tilde{\chi}_i^0, \tilde{\ell}_R\tilde{\ell}_R)$: To obtain these quantities one has to exchange in (70) and (71)

$$\Delta^{t_i}(\tilde{\ell}_L) \to \Delta^{t_i}(\tilde{\ell}_R), \quad \Delta^{u_i}(\tilde{\ell}_L) \to \Delta^{u_i}(\tilde{\ell}_R), \quad O_{ki}^{''L} \to O_{ki}^{''R}, \quad f_{\ell i}^L \to f_{\ell i}^R, \quad L_\ell \to R_\ell.$$

The expressions $D_j(\tilde{\chi}_j^0)$, (15), for the decay $\tilde{\chi}_j^0(p_4) \to \tilde{\chi}_l^0(p_8) + \ell^+(p_9) + \ell^-(p_{10})$ and the corresponding scalar products are obtained by the following substitutions in (69)–(74):

$$p_5 \to p_8, p_6 \to p_9, p_7 \to p_{10}, \quad m_i \to m_j, m_k \to m_l, \eta_i \to \eta_j, \eta_k \to \eta_l, \quad (75)$$
$$O_{ki}^L \to O_{lj}^L, \quad O_{ki}^R \to O_{lj}^R, \quad (76)$$
$$\Delta^{s_i}(Z) \to \Delta^{s_j}(Z), \quad \Delta^{t_i}(\tilde{\ell}_{L,R}) \to \Delta^{t_j}(\tilde{\ell}_{L,R}), \quad \Delta^{u_i}(\tilde{\ell}_{L,R}) \to \Delta^{u_j}(\tilde{\ell}_{L,R}). \quad (77)$$

4.2 Neutralino polarization dependent quantities

We first give $\Sigma_D^a(\tilde{\chi}_i^0)$ of (15) which contains the polarization vector $s^a(\tilde{\chi}_i^0)$:

$$\Sigma_D^a(\tilde{\chi}_i^0) = \Sigma_D^a(\tilde{\chi}_i^0, ZZ) + \Sigma_D^a(\tilde{\chi}_i^0, Z\tilde{\ell}_L) + \Sigma_D^a(\tilde{\chi}_i^0, Z\tilde{\ell}_R) + \Sigma_D^a(\tilde{\chi}_i^0, \tilde{\ell}_L\tilde{\ell}_L) + \Sigma_D^a(\tilde{\chi}_i^0, \tilde{\ell}_R\tilde{\ell}_R). \quad (78)$$

The analytical expressions for $\Sigma_D^a(\tilde{\chi}_i^0)$, (78), read:

$$\Sigma_D^a(\tilde{\chi}_i^0, ZZ) = 8\frac{g^4}{\cos^4\Theta_W}|\Delta^{s_i}(Z)|^2(R_\ell^2 - L_\ell^2)\Big[-[(ReO_{ki}^{''L})^2 - (ImO_{ki}^{''L})^2]g_3^a + |O_{ki}^{''L}|^2(g_1^a - g_2^a)\Big], \quad (79)$$

$$\Sigma_D^a(\tilde{\chi}_i^0, Z\tilde{\ell}_L) = \frac{4g^4}{\cos^2\Theta_W}L_\ell Re\Big\{\Delta^{s_i}(Z)\Big[f_{\ell i}^L f_{\ell k}^{L*}\Delta^{t_i*}(\tilde{\ell}_L) \times\big(-2O_{ki}^{''L}g_1^a + O_{ki}^{''L*}(g_3^a - g_4^a)\big) + f_{\ell i}^{L*}f_{\ell k}^L\Delta^{u_i*}(\tilde{\ell}_L)\big(2O_{ki}^{''L*}g_2^a + O_{ki}^{''L}(g_3^a - g_4^a)\big)\Big]\Big\}, \quad (80)$$

$$\Sigma_D^a(\tilde{\chi}_i^0, \tilde{\ell}_L\tilde{\ell}_L) = 2g^4\Big[|f_{\ell i}^L|^2|f_{\ell k}^L|^2[|\Delta^{u_i}(\tilde{\ell}_L)|^2 g_2^a - |\Delta^{t_i}(\tilde{\ell}_L)|^2 g_1^a] + Re\big\{(f_{\ell i}^{L*})^2(f_{\ell k}^L)^2\Delta^{t_i}(\tilde{\ell}_L)\Delta^{u_i*}(\tilde{\ell}_L)(g_3^a + g_4^a)\big\}\Big], \quad (81)$$

where we have introduced the following abbrevations involving the polarization vector $s^a(\tilde{\chi}_i^0)$, (26)–(28), with $a = 1, 2, 3$:

$$g_1^a = \eta_i m_i(p_5p_7)(p_6 s^a(\tilde{\chi}_i^0)), \quad (82)$$
$$g_2^a = \eta_i m_i(p_5p_6)(p_7 s^a(\tilde{\chi}_i^0)), \quad (83)$$
$$g_3^a = \eta_k m_k[(p_3p_6)(p_7 s^a(\tilde{\chi}_i^0)) - (p_3p_7)(p_6 s^a(\tilde{\chi}_i^0))], \quad (84)$$
$$g_4^a = i\eta_k m_k\epsilon_{\mu\nu\rho\tau}s^{a\mu}(\tilde{\chi}_i^0)p_3^\nu p_7^\rho p_6^\tau. \quad (85)$$

$\Sigma_D^a(\tilde{\chi}_i^0, Z\tilde{\ell}_R)$, $\Sigma_D^a(\tilde{\chi}_i^0, \tilde{\ell}_R\tilde{\ell}_R)$: To obtain these quantities one has to exchange in (80) and (81)

$$\Delta^{t_i}(\tilde{\ell}_L) \to \Delta^{t_i}(\tilde{\ell}_R), \quad \Delta^{u_i}(\tilde{\ell}_L) \to \Delta^{u_i}(\tilde{\ell}_R), \quad O_{ki}^{''L} \to O_{ki}^{''R}, \quad f_{\ell i}^L \to f_{\ell i}^R, \quad L_\ell \to R_\ell.$$

In addition, one has to change the sign of g_1^a, g_2^a, g_3^a, but not of g_4^a.

The expression g_4^a can be expanded in triple product correlations which are sensitive to the component of the spin vector perpendicular to the scattering plane.

The corresponding expressions $\Sigma_D^b(\tilde{\chi}_j^0)$, (15), for the decay $\tilde{\chi}_j^0(p_4) \to \tilde{\chi}_l^0(p_8) + \ell^+(p_9) + \ell^-(p_{10})$ are obtained by the same substitutions as (75)–(77), and the additional substitution $s^a(\tilde{\chi}_i^0) \to s^b(\tilde{\chi}_j^0)$ in (82)–(85).

5 Numerical results

In the following numerical analysis we study $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ with $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 e^+e^-$ for various polarizations of the e^- beam. The calculations are done in the MSSM. We take the parameters M', M, μ, $\tan\beta$ real. Since we want to study the influence of the parameter M' we do not use a relation between M' and M. We will also study the dependence on the selectron masses $m_{\tilde{e}_L}$, $m_{\tilde{e}_R}$.

We shall choose three different examples of parameter sets. In all these examples we choose $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, and vary M' between 40 GeV and 160

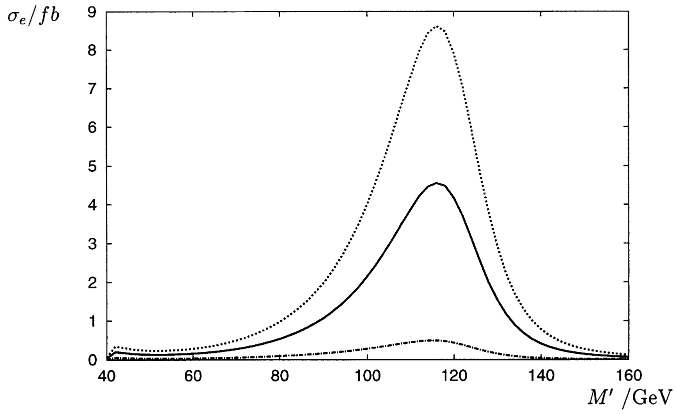

Fig. 2. M' dependence of the cross section σ_e (defined in (86)) near threshold ($\sqrt{s} = m_{\tilde{\chi}^0_1} + m_{\tilde{\chi}^0_2} + 50$ GeV) with $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, $m_{\tilde{e}_L} = 1000$ GeV, and $m_{\tilde{e}_R} = 200$ GeV for the three cases: unpolarized beams (*solid line*), e^- beam polarized, $P^3_- = +90\%$ (*dotted line*) and $P^3_- = -90\%$ (*dash-dotted line*).

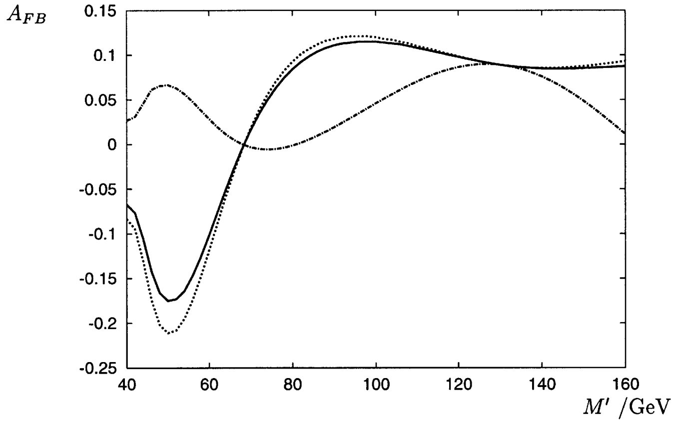

Fig. 3. M' dependence of the lepton forward–backward asymmetry A_{FB} near threshold ($\sqrt{s} = m_{\tilde{\chi}^0_1} + m_{\tilde{\chi}^0_2} + 50$ GeV) with $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, $m_{\tilde{e}_L} = 1000$ GeV, and $m_{\tilde{e}_R} = 200$ GeV for the three cases: unpolarized beams (*solid line*), e^- beam polarized, $P^3_- = +90\%$ (*dotted line*) and $P^3_- = -90\%$ (*dash-dotted line*).

GeV. Especially the mass of the $\tilde{\chi}^0_1$ is very sensitive to M'. For these parameters $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$ are dominantly gauginos and have small couplings to Z^0. For the selectron masses we take

i) $m_{\tilde{e}_L} = 1000$ GeV, $m_{\tilde{e}_R} = 200$ GeV;
ii) $m_{\tilde{e}_L} = 200$ GeV, $m_{\tilde{e}_R} = 1000$ GeV;
iii) $m_{\tilde{e}_L} = 176$ GeV, $m_{\tilde{e}_R} = 161$ GeV.

In i) and ii) we want to study the influence of $\tilde{e}_L$ and $\tilde{e}_R$ exchange for large selectron mass splitting. Scenario ii) with $m_{\tilde{e}_R} > m_{\tilde{e}_L}$ may be realized in extended SUSY models [20]. For $M' = 78.7$ GeV, example iii) corresponds to the mSUGRA scenario studied in [21], except for $m_{\tilde{e}_R}$ which in our case is set equal to $m_{\tilde{\nu}}$. In iii) we want to study the case of small selectron mass splitting.

We present results for the cross section

$$\sigma_e = \sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2)BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 e^+e^-), \tag{86}$$

and the forward–backward asymmetry

$$A_{FB} = \frac{\sigma_e(\cos\Theta_- > 0) - \sigma_e(\cos\Theta_- < 0)}{\sigma_e(\cos\Theta_- > 0) + \sigma_e(\cos\Theta_- < 0)} \tag{87}$$

of the electron from the decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 e^+e^-$. In (87) Θ_- is the angle between the incoming electron beam and the outgoing e^-.

The forward–backward asymmetry A_{FB} is largest near the production threshold. We therefore study in all three examples σ_e and A_{FB} at $\sqrt{s} = m_{\tilde{\chi}^0_1} + m_{\tilde{\chi}^0_2} + 50$ GeV, and in example iii) also at $\sqrt{s} = 500$ GeV. As for the polarization of the e^- beam we take $P^3_- = \pm 90\%$.

5.1 Cross sections

We first study the M' dependence of $\sigma_e = \sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2)BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 e^+e^-)$ near the production threshold ($\sqrt{s} = m_{\tilde{\chi}^0_1} + m_{\tilde{\chi}^0_2} + 50$ GeV). We begin with case i), where $\tilde{e}_L$ exchange is suppressed. Figure 2 shows the corresponding M' dependence for unpolarized beams and for the e^- beam polarizations $P^3_- = +90\%$ and $P^3_- = -90\%$, with M, μ and $\tan\beta$ as given above. Clearly, a right polarized e^- beam yields the largest cross section because it enhances the $\tilde{e}_R$ exchange. The production cross section for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ has a maximum at $M' \approx 130$ GeV, where also the $\tilde{e}_R$ exchange contribution is maximal. The cross section σ_e, (86), has its maximum at $M' \approx 118$ GeV. This shift is due to the fact that the leptonic decay branching ratio of $\tilde{\chi}^0_2$ has a maximum at $M' \approx 118$ GeV and then strongly decreases.

Obviously, the characters of $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$ change with varying M'. With increasing M' the $\tilde{B}$ component of $\tilde{\chi}^0_1$ decreases and the W^3-ino and the higgsino components increase. The opposite is true for $\tilde{\chi}^0_2$. The Z^0 couplings are small and almost constant up to $M' \approx 120$ GeV, $O''^L_{12} \approx 0.015$, and decrease for larger M'. The product of the $\tilde{e}_R$ couplings, $|f^R_{e1} f^R_{e2}|$, has a maximum at $M' \approx 130$ GeV.

We compare this with case ii), where $\tilde{e}_R$ exchange is suppressed. Figure 4 shows the corresponding M' dependence. Now a left polarized e^- beam leads to the largest cross section because the $\tilde{e}_L$ exchange is favoured. There is a maximum at $M' \approx 60$ GeV and a minimum at $M' \approx 120$ GeV. The maximum at $M' \approx 60$ GeV can be explained by a corresponding maximum of the leptonic branching ratio. The minimum at $M' \approx 120$ GeV is due to the vanishing of $e\tilde{e}_L\tilde{\chi}^0_1$ coupling f^L_{e1} at this value of M'.

In example iii) the mass difference between $\tilde{e}_L$ and $\tilde{e}_R$ is small. Therefore, $\tilde{e}_L$ and $\tilde{e}_R$ exchange contribute. We show in Figs. 6 and 8 the M' dependence for this case near threshold and at $\sqrt{s} = 500$ GeV, respectively. For right polarized e^- beams the cross section behaviour is similar to that of case i), and for left polarized e^- beams it is similar to that of case ii). At $\sqrt{s} = 500$ GeV the cross section is about a factor 2 bigger than near threshold but

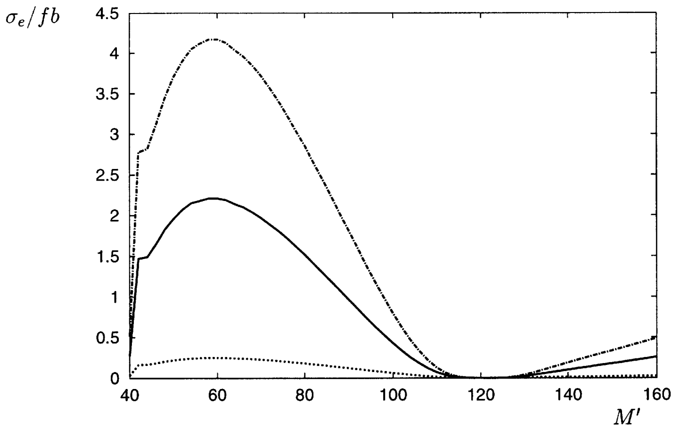

Fig. 4. M' dependence of the cross section σ_e (defined in (86)) near threshold ($\sqrt{s} = m_{\tilde{\chi}^0_1} + m_{\tilde{\chi}^0_2} + 50$ GeV) with $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, $m_{\tilde{e}_L} = 200$ GeV, and $m_{\tilde{e}_R} = 1000$ GeV for the three cases: unpolarized beams (*solid line*), e^- beam polarized, $P^3_- = +90\%$ (*dotted line*) and $P^3_- = -90\%$ (*dash-dotted line*).

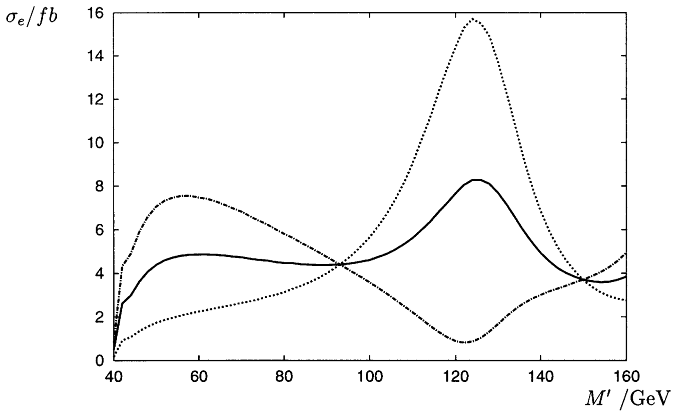

Fig. 6. M' dependence of the cross section σ_e (defined in (86)) near threshold ($\sqrt{s} = m_{\tilde{\chi}^0_1} + m_{\tilde{\chi}^0_2} + 50$ GeV) with $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, $m_{\tilde{e}_L} = 176$ GeV, and $m_{\tilde{e}_R} = 161$ GeV for the three cases: unpolarized beams (*solid line*), e^- beam polarized, $P^3_- = +90\%$ (*dotted line*) and $P^3_- = -90\%$ (*dash-dotted line*).

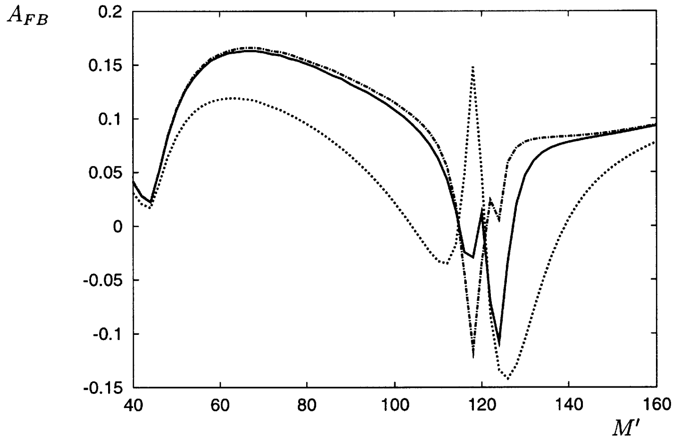

Fig. 5. M' dependence of the lepton forward–backward asymmetry A_{FB} near threshold ($\sqrt{s} = m_{\tilde{\chi}^0_1} + m_{\tilde{\chi}^0_2} + 50$ GeV) with $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, $m_{\tilde{e}_L} = 200$ GeV, and $m_{\tilde{e}_R} = 1000$ GeV for the three cases: unpolarized beams (*solid line*), e^- beam polarized, $P^3_- = +90\%$ (*dotted line*) and $P^3_- = -90\%$ (*dash-dotted line*).

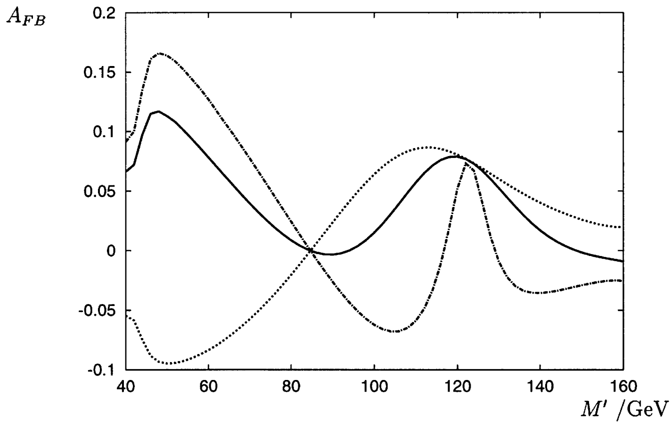

Fig. 7. M' dependence of the lepton forward–backward asymmetry A_{FB} near threshold ($\sqrt{s} = m_{\tilde{\chi}^0_1} + m_{\tilde{\chi}^0_2} + 50$ GeV) with $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, $m_{\tilde{e}_L} = 176$ GeV, and $m_{\tilde{e}_R} = 161$ GeV for the three cases: unpolarized beams (*solid line*), e^- beam polarized, $P^3_- = +90\%$ (*dotted line*) and $P^3_- = -90\%$ (*dash-dotted line*).

has a similar M' dependence. In all cases there is a small step at about $M' = 42 - 44$ GeV, which is due to the opening of the two-body decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 Z^0$.

5.2 Lepton forward–backward asymmetries

In this subsection we study the M' dependence of the forward–backward asymmetry A_{FB} of the decay electron e^-, as defined in (87). The decay electron angular distributions and the corresponding forward–backward asymmetry are very sensitive to the spin correlations $\Sigma^a_P \Sigma^a_D$, ($\Sigma^b_P \Sigma^b_D$), (15), and are the result of a complex interplay between production and decay. As the spin correlations between production and decay are strongest near threshold, the forward–backward asymmetry will also be largest there.

We show in Figs. 3, 5, and 7 A_{FB} near threshold as a function of M' for the cases i), ii), and iii), respectively. As can be seen A_{FB} is very sensitive to the masses of $\tilde{e}_L$ and $\tilde{e}_R$, and the mass splitting between them. In all cases A_{FB} has a pronounced M' dependence. The selectron couplings f^L_{ei} and f^R_{ei}, i =1, 2, exhibit a characteristic M' dependence, which is reflected in the M' behaviour of A_{FB}. Moreover, by choosing different e^- beam polarizations the $\tilde{e}_L$ and $\tilde{e}_R$ contributions can be enhanced or suppressed.

The small dip of the asymmetry at $M' = 42 - 44$ GeV is due to the opening of the two-body decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 Z^0$.

The behaviour of A_{FB} in Fig. 5 at about $M' = 115 - 125$ GeV is due to the vanishing of $e\tilde{e}_L\tilde{\chi}^0_1$ coupling f^L_{e1} at $M' \approx 120$ GeV and a complicated interplay between the

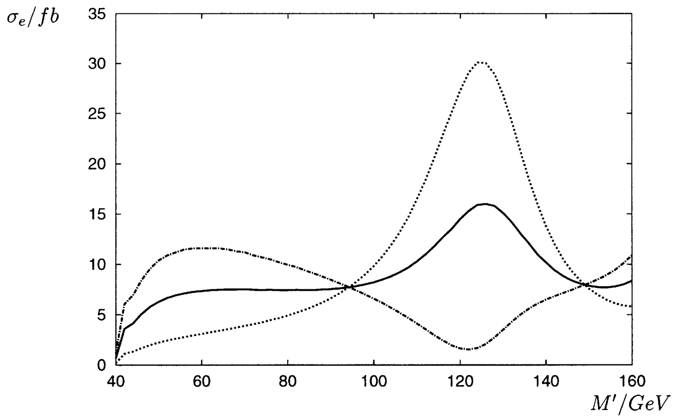

Fig. 8. M' dependence of the cross section σ_e (defined in (86)) at $\sqrt{s} = 500$ GeV with $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, $m_{\tilde{e}_L} = 176$ GeV, and $m_{\tilde{e}_R} = 161$ GeV for the three cases: unpolarized beams (*solid line*), e^- beam polarized, $P^3_- = +90\%$ (*dotted line*) and $P^3_- = -90\%$ (*dash-dotted line*).

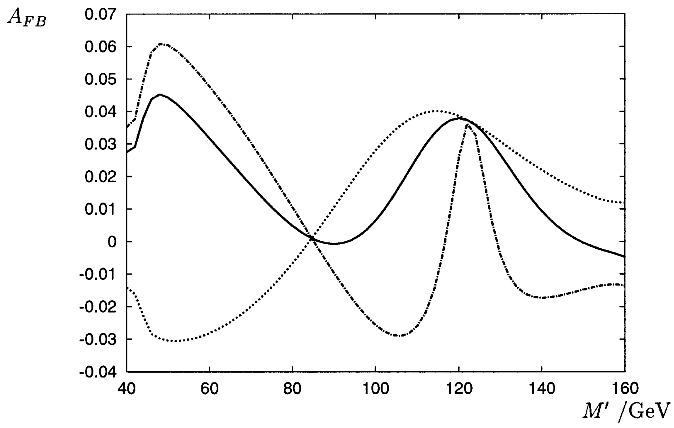

Fig. 9. M' dependence of the lepton forward–backward asymmetry A_{FB} at $\sqrt{s} = 500$ GeV with $M = 152$ GeV, $\mu = 316$ GeV, $\tan\beta = 3$, $m_{\tilde{e}_L} = 176$ GeV, and $m_{\tilde{e}_R} = 161$ GeV for the three cases: unpolarized beams (*solid line*), e^- beam polarized, $P^3_- = +90\%$ (*dotted line*) and $P^3_- = -90\%$ (*dash-dotted line*).

Z^0, $\tilde{e}_L$ and $\tilde{e}_R$ contributions, which are all very small (see Fig. 4).

In Fig. 9 we show the M' dependence of A_{FB} at $\sqrt{s} = 500$ GeV for case iii). This is very similar to that near threshold, Fig. 7, but the magnitude is smaller by a factor 2 to 3, because with increasing $\sqrt{s}$ the spin correlations decrease.

A numerical analysis for both beams polarized has been given in [11] and will be continued in [22].

6 Summary

We have given the full analytical expressions for the differential cross section for $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_j$ with polarized beams and the subsequent leptonic decays $\tilde{\chi}^0_i \to \ell^+\ell^-\tilde{\chi}^0_k$ and $\tilde{\chi}^0_j \to \ell^+\ell^-\tilde{\chi}^0_l$, taking into account the complete spin correlations between production and decay. The production spin density matrix is presented in the laboratory system. The formulae for the decay processes are written covariantly involving explicitly the neutralino polarization vectors. When combining the production and decay process the polarization vectors in the laboratory system as given in (26)–(31) have to be taken.

We have presented numerical results for the cross section and the lepton forward–backward asymmetry for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$, $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 e^+e^-$. We have studied the dependence on the parameter M' for various mass splittings between $\tilde{e}_L$ and $\tilde{e}_R$ and different e^- beam polarizations.

The cross section σ_e shows a characteristic dependence on M' and the masses of the exchanged selectrons as well as on the beam polarization.

The lepton forward–backward asymmetry A_{FB} can only be explained by the presence of spin correlations between production and decay, as it would be zero in the production process alone. A_{FB} depends very sensitively on the SUSY parameters and the beam polarizations. Therefore, this quantity is an additional useful observable for a more precise determination of the parameters. Different beam polarizations help disentangle the contribution from $\tilde{e}_L$ and $\tilde{e}_R$ exchange.

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Appendices

A Amplitudes

We give the helicity amplitudes $T_P^{\lambda_i\lambda_j}(\alpha,\beta)$ for production and $T_{D,\lambda_i}(\alpha,\beta)$, $T_{D,\lambda_j}(\alpha,\beta)$ for the decays, corresponding to the Feynman diagrams in Fig. 1 (α denotes the channel, β denotes the exchanged particle).

The amplitudes $T_P^{\lambda_i\lambda_j}(\alpha,\beta)$ for the production, $e^-(p_1)$ $e^+(p_2) \to \tilde{\chi}^0_i(p_3)\tilde{\chi}^0_j(p_4)$ read:

$$T_P^{\lambda_i\lambda_j}(s,Z) = \frac{g^2}{\cos^2\Theta_W}\Delta^s(Z)\bar{v}(p_2)\gamma^\mu(L_\ell P_L + R_\ell P_R)u(p_1) \times\bar{u}(p_4,\lambda_j)\gamma_\mu(O''^L_{ji}P_L + O''^R_{ji}P_R) \times v(p_3,\lambda_i), \quad (A.1)$$

$$T_P^{\lambda_i\lambda_j}(t,\tilde{\ell}_L) = -g^2 f^L_{\ell i} f^{L*}_{\ell j}\Delta^t(\tilde{\ell}_L)\bar{v}(p_2)P_R v(p_3,\lambda_i) \times\bar{u}(p_4,\lambda_j)P_L u(p_1), \quad (A.2)$$

$$T_P^{\lambda_i\lambda_j}(t,\tilde{\ell}_R) = -g^2 f^R_{\ell i} f^{R*}_{\ell j}\Delta^t(\tilde{\ell}_R)\bar{v}(p_2)P_L v(p_3,\lambda_i) \times\bar{u}(p_4,\lambda_j)P_R u(p_1), \quad (A.3)$$

$$T_P^{\lambda_i\lambda_j}(u,\tilde{\ell}_L) = g^2 f_{\ell i}^{L*} f_{\ell j}^{L} \Delta^u(\tilde{\ell}_L)\bar{v}(p_2)P_R v(p_4,\lambda_j) \times \bar{u}(p_3,\lambda_i)P_L u(p_1), \quad (A.4)$$

$$T_P^{\lambda_i\lambda_j}(u,\tilde{\ell}_R) = g^2 f_{\ell i}^{R*} f_{\ell j}^{R} \Delta^u(\tilde{\ell}_R)\bar{v}(p_2)P_L v(p_4,\lambda_j) \times \bar{u}(p_3,\lambda_i)P_R u(p_1). \quad (A.5)$$

The amplitudes $T_{D,\lambda_i}(\alpha,\beta)$ for the decay of the $\tilde{\chi}_i^0(p_3) \to \tilde{\chi}_k^0(p_5)\ell^+(p_6)\ell^-(p_7)$ read:

$$T_{D,\lambda_i}(s_i) = -\frac{g^2}{\cos^2\Theta_W}\Delta^{s_i}(Z)\bar{u}(p_7)\gamma^\mu(L_\ell P_L + R_\ell P_R) \times v(p_6)\bar{u}(p_5)\gamma_\mu(O_{ki}^{''L}P_L + O_{ki}^{''R}P_R) \times u(p_3,\lambda_i), \quad (A.6)$$

$$T_{D,\lambda_i}(t_i,\tilde{\ell}_L) = -g^2 f_{\ell k}^{L} f_{\ell i}^{L*}\Delta^{t_i}(\tilde{\ell}_L)\bar{u}(p_7)P_R v(p_5) \times \bar{v}(p_3,\lambda_i)P_L v(p_6), \quad (A.7)$$

$$T_{D,\lambda_i}(t_i,\tilde{\ell}_R) = -g^2 f_{\ell k}^{R} f_{\ell i}^{R*}\Delta^{t_i}(\tilde{\ell}_R)\bar{u}(p_7)P_L v(p_5) \times \bar{v}(p_3,\lambda_i)P_R v(p_6), \quad (A.8)$$

$$T_{D,\lambda_i}(u_i,\tilde{\ell}_L) = +g^2 f_{\ell i}^{L} f_{\ell k}^{L*}\Delta^{u_i}(\tilde{\ell}_L)\bar{u}(p_7)P_R u(p_3,\lambda_i) \times \bar{u}(p_5)P_L v(p_6), \quad (A.9)$$

$$T_{D,\lambda_i}(u_i,\tilde{\ell}_R) = +g^2 f_{\ell i}^{R} f_{\ell k}^{R*}\Delta^{u_i}(\tilde{\ell}_R)\bar{u}(p_7)P_L u(p_3,\lambda_i) \times \bar{u}(p_5)P_R v(p_6). \quad (A.10)$$

The corresponding amplitudes $T_{D,\lambda_j}(\alpha,\beta)$ for the decay of the $\tilde{\chi}_j^0(p_4) \to \tilde{\chi}_l^0(p_8)\ell^+(p_9)\ell^-(p_{10})$ are obtained by exchanging in (A.6)–(A.10):

$$s_i \to s_j, t_i \to t_j, u_i \to u_j, \quad \Delta^{s_i} \to \Delta^{s_j}, \Delta^{t_i} \to \Delta^{t_j}, \Delta^{u_i} \to \Delta^{u_j}, \quad (A.11)$$

$$p_5 \to p_8, p_6 \to p_9, p_7 \to p_{10}, O_{ki}^L \to O_{lj}^L, O_{ki}^R \to O_{lj}^R. \quad (A.12)$$

B Spin formalism

The amplitude for the whole process, (13), is

$$T = \Delta(\tilde{\chi}_i^+)\Delta(\tilde{\chi}_j^-)\sum_{\lambda_i,\lambda_j} T_P^{\lambda_i\lambda_j} T_{D,\lambda_i} T_{D,\lambda_j}, \quad (B.1)$$

with the helicity amplitude $T_P^{\lambda_i\lambda_j}$ for the production process and T_{D,λ_i}, T_{D,λ_j} for the decay processes, and the propagators $\Delta(\tilde{\chi}_{i,j}^\pm) = 1/[p_{i,j}^2 - m_{i,j}^2 + i m_{i,j}\Gamma_{i,j}]$. Here $\lambda_{i,j}, p_{i,j}^2$, $m_{i,j}$, $\Gamma_{i,j}$ denote the helicity, four–momentum squared, mass and width of $\tilde{\chi}_{i,j}^\pm$. The amplitude squared

$$|T|^2 = |\Delta(\tilde{\chi}_i^+)|^2|\Delta(\tilde{\chi}_j^-)|^2 \rho_P^{\lambda_i\lambda_j\lambda_i'\lambda_j'}\rho_{D,\lambda_i'\lambda_i}\rho_{D,\lambda_j'\lambda_j} \quad (B.2)$$

(sum convention used) is thus composed of the (unnormalized) spin density production matrix

$$\rho_P^{\lambda_i\lambda_j\lambda_i'\lambda_j'} = T_P^{\lambda_i\lambda_j}T_P^{\lambda_i'\lambda_j'*} \quad (B.3)$$

of $\tilde{\chi}_{i,j}^0$ and the decay matrices

$$\rho_{D,\lambda_i'\lambda_i} = T_{D,\lambda_i}T_{D,\lambda_i'}^* \text{ and } \rho_{D,\lambda_j'\lambda_j} = T_{D,\lambda_j}T_{D,\lambda_j'}^*. \quad (B.4)$$

Introducing a suitable set of polarization vectors for each of the neutralinos one can expand the spin density matrix of the production process and the decay matrices of both neutralinos in terms of Pauli matrices.

The spin density production matrix reads:

$$\rho_P^{\lambda_i\lambda_j\lambda_i'\lambda_j'} = \delta_{\lambda_i\lambda_i'}\delta_{\lambda_j\lambda_j'}P + \delta_{\lambda_j\lambda_j'}\sum_a \sigma_{\lambda_i\lambda_i'}^a \Sigma_P^a(\tilde{\chi}_i^0) + \delta_{\lambda_i\lambda_i'}\sum_b \sigma_{\lambda_j\lambda_j'}^b \Sigma_P^b(\tilde{\chi}_j^0) + \sum_{ab}\sigma_{\lambda_i\lambda_i'}^a\sigma_{\lambda_j\lambda_j'}^b \Sigma_P^{ab}(\tilde{\chi}_i^0\tilde{\chi}_j^0), \quad (B.5)$$

and the matrices for the decays read:

$$\rho_{D,\lambda_i'\lambda_i} = \delta_{\lambda_i'\lambda_i}D(\tilde{\chi}_i^0) + \sum_a \sigma_{\lambda_i'\lambda_i}^a \Sigma_D^a(\tilde{\chi}_i^0), \quad (B.6)$$

$$\rho_{D,\lambda_j'\lambda_j} = \delta_{\lambda_j'\lambda_j}D(\tilde{\chi}_j^0) + \sum_b \sigma_{\lambda_j'\lambda_j}^b \Sigma_D^b(\tilde{\chi}_j^0). \quad (B.7)$$

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